b) $\forall n \geq 2 ;\ S_n = 1 + \frac{5}{3^2} + \frac{7}{3^3} + \dots + \frac{2n+1}{3^n}$

$$= \sum_{k=1}^{n} U_k$$

$\forall k \in \mathbb{N} ;\ V_k = U_{k+1} - \frac{1}{3} U_k$

Donc $\sum_{k=1}^{n-1} V_k = \sum_{k=1}^{n-1} U_{k+1} - \frac{1}{3} \sum_{k=1}^{n-1} U_k$

$$= \sum_{k=2}^{n} U_k - \frac{1}{3} \sum_{k=1}^{n-1} U_k$$

$$= \sum_{k=2}^{n} U_k - \frac{1}{3} \left(\sum_{k=1}^{n} U_k - U_n \right)$$

$$= -U_1 + \sum_{k=1}^{n} U_k - \frac{1}{3} S_n + \frac{1}{3} U_n$$

$$= -U_1 + S_n - \frac{1}{3} S_n + \frac{1}{3} U_n$$

$$= \frac{2}{3} S_n + \frac{1}{3} U_n - U_1$$

Donc $\sum_{k=1}^{n-1} V_k = \frac{2}{3} S_n + \frac{1}{3} U_n - U_1$

$\Rightarrow S_n = \frac{3}{2} \left(\sum_{k=1}^{n-1} V_k - \frac{1}{3} U_n + U_1 \right)$

$\Rightarrow S_n = \frac{3}{2} \left(\frac{1}{3} \left(1 - \frac{1}{3^{n-1}} \right) - \frac{1}{3} \frac{2n+1}{3^n} + 1 \right)$